Jet Noise Source Distribution from Far-Field Cross Correlations

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Theme

THIS paper contains the development of a technique to determine the relationship between the unknown source correlation function and the correlation of the far-field acoustic pressure. Based on Lighthill's equivalent source model, a Fredholm integral equation relating the unknown source-correlation function to the far-field pressure correlation is derived. Analytically, this is a statistically inverse radiation problem. It is assumed that the source is distributed over an axisymmetric region, that is, a cylindrical region in an axisymmetric jet.

This technique was developed for an application in the study of the installation effects for static and moving jets, since the spatial and temporal distribution of the source relative to the jet were required.[1] In the literature, several techniques are available, for example, Refs. 2 through 5, which provide information of the spatial distribution only or suitable to reproduce far-field intensity without body effects. In addition, further information can be extracted from the source field if obtained from the far-field two-point correlation.[6,7]

Contents

Consider the propagation of an acoustic wave generated by turbulence in the jet confined in a region D in space, Fig. 1; the sound pressure in the spectral or frequency domain satisfies

$$(\nabla^2 + k^2)\hat{p}(x,\omega,\alpha) = \hat{q}(\vec{y},\omega,\alpha) \tag{1}$$

where $\hat{q}(\vec{y},w,\alpha)$ is the random source term depending on a stochastic parameter α, or a sample point. Equation (1) is subject to a radiation condition. The solution of Eq. (1) is known to be

$$\hat{p}(\vec{x},\omega,\alpha) = \int_D \frac{e^{ikr}}{4\pi r}\hat{q}(\vec{y},\omega,\alpha)\,d\vec{y} \tag{2}$$

Let

$$\hat{Q}(\vec{y}',\vec{y}'',\omega) = \langle \hat{q}(\vec{y}',\omega)\hat{q}^*(\vec{y}'',\omega)\rangle$$

and

$$\hat{R}_{pp}(\hat{x}_1,\hat{x}_2,\omega) = \langle \hat{p}(\vec{x}_2,\omega)\hat{p}^*(\vec{x}_1,\omega)\rangle$$

$$= \left(\frac{1}{4\pi}\right)^2 \int_D\int_D \frac{e^{ik(r'-r'')}}{r'\cdot r''}\hat{Q}(\vec{y}',\vec{y}'',\omega)\,d\vec{y}'\,d\vec{y}'' \tag{3}$$

which relates the source correlation to the pressure correlation in the spectral domain.

The problem of interest is to determine the source correlation function $\hat{q}$ given a partial or incomplete set of data of $\hat{R}_{pp}$ on a surface S or a region Ω in the space. Since the far-field surface is located far away from the source region, the following asymptotic relation is obtained

$$\hat{R}_{pp}(\vec{x}_1,\vec{x}_2,\omega) \sim G(\xi)G^*(\xi)\int_D\int_D \hat{Q}(\vec{y}',\vec{y}'',\omega)$$

$$e^{-ik(\hat{e}_1\cdot\vec{y}-\hat{e}_2\cdot\vec{y}'')}\,d\vec{y}'\,d\vec{y}'' \tag{4}$$

where $G(\xi)$ is the free space Green's function

$$G(\xi) = e^{ik\xi}/4\pi\xi$$

and $\xi = |\vec{x}| \gg |\vec{y}|$, $\hat{e}_i = \vec{x}_i/|\vec{x}_i|$. Since GG^* on the right-hand side of Eq. (4) represents the radiation field due to a point source, the far-field correlation function

$$A(\hat{e}_1,\hat{e}_2,\omega) = \lim_{\xi',\xi''\to\infty} \frac{\hat{R}_{pp}(\vec{x}_1,\vec{x}_2,\omega)}{G(\xi')G^*(\xi'')}$$

$$= \int_D\int_D \hat{Q}(\vec{y}',\vec{y}'',\omega)e^{ik(\hat{e}_1\vec{y}'-\hat{e}_2\cdot\vec{y}'')}\,d\vec{y}'\,d\vec{y}'' \tag{5}$$

The above equation is an integral equation for the source correlation $\hat{Q}$.

Let an axisymmetric jet be confined in a cylindrical region D with radius a and length L. The cylindrical coordinates system (ρ,ϕ,z) is shown in Fig. 2. For an axisymmetric source, the integral equation (5) in the cylindrical coordinates becomes

$$A(\theta_1,\theta_2,\omega) = \int_0^L\int_0^L \hat{Q}_0(z',z'',\omega)$$

$$\exp k(z'\cos\theta_1 - z''\cos\theta_2)\,dz'\,dz'' \tag{6}$$

The experimental far-field cross correlation measured in a broadband at two fixed positions $\theta = 30°$ and $90°$ are shown in Figs. 3 and 4. More detailed information and additional measurements of the correlation over the far-field sphere can be found in Ref. 1.

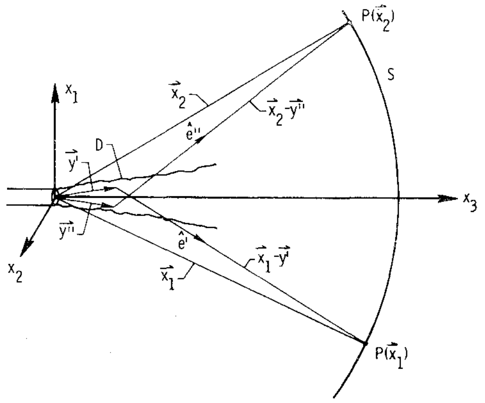

Fig. 1 Source far-field coordinate.

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Index categories: Noise; Aeroacoustics.

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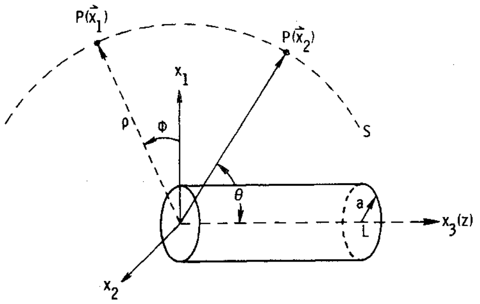

Fig. 2 Coordinate system.

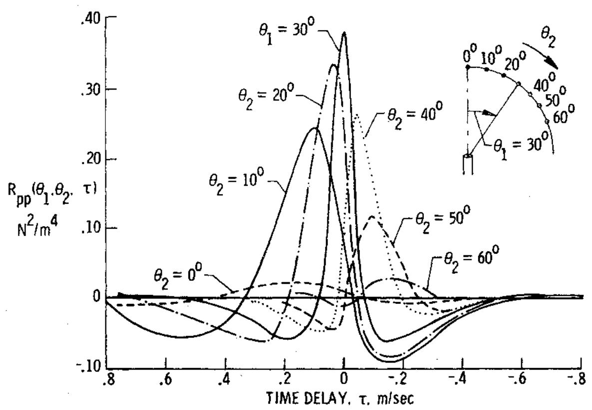

Fig. 3 Broadband far-field space-time correlation; $\theta_1 = 30°$.

Let the real part of A in Eq. (6) be the far-field correlation amplitude denoted by I

$$I(\theta_1,\theta_2,\omega)=Re\{A(\theta_1,\theta_2,\omega)\}=\int_0^L\int_0^L \hat{Q}_0(z',\tau'',\omega)$$

$$\cos k(z'\cos\theta_1 - z''\cos\theta_2)\,dz'dz'' \qquad (7)$$

The above integral equation frequently cannot be solved analytically for the unknown source correlation function $\hat{Q}_0$. Let

$$\tilde{Q}_0(\vec{z},\omega)=\sum_{\ell,m=1}^{N} \tilde{q}_{\ell,m}\psi_\ell(z)\psi_m(z') \qquad (8)$$

where $\vec{z}=(z,z')$, ψ_ℓ and ψ_m are piecewise linear functions in z and z', respectively. Upon substituting Eq. (8) into Eq. (7) a system of linear algebraic equations is obtained.

$$I_j(\vec{\phi},\omega)=\sum_{\ell,m=1}^{N} c_{j,\ell m}\tilde{q}_{\ell,m} \qquad j=1,\ldots,M \qquad (9)$$

where $I_j(\vec{\phi},\omega)=I(\vec{\phi}_j,\omega)$, $\vec{\phi}=(\phi,\phi')$, M is the number of measurements ($M\geq N$),

$$c_{j,\ell m}=\int_0^L\int_0^L \psi_\ell(z)\psi_m(z')K(\vec{z},\vec{\phi}_j,k)\,dzdz', k=\omega/c \qquad (10)$$

and the kernel $K(\vec{z},\vec{\phi},k)=\cos k(z\cos\phi - z'\cos\phi')$. The linear equation (9) can be solved to obtain a least-square solution $\tilde{q}_{\ell,m}$. The method for stabilizing a solution for the linear system may be found in Refs. 7 and 8. To test the validity of the numerical scheme, a numerical experiment was reported in Ref. 1.

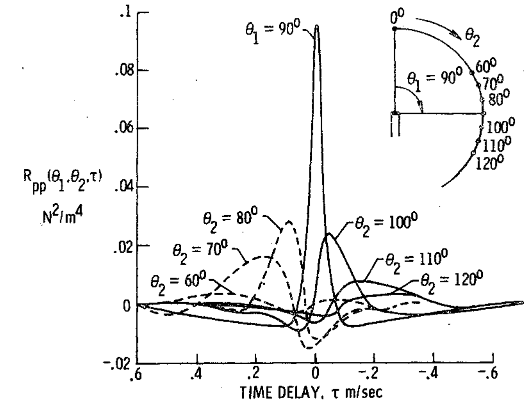

Fig. 4 Broadband far-field space-time correlation; $\theta_1 = 90°$.

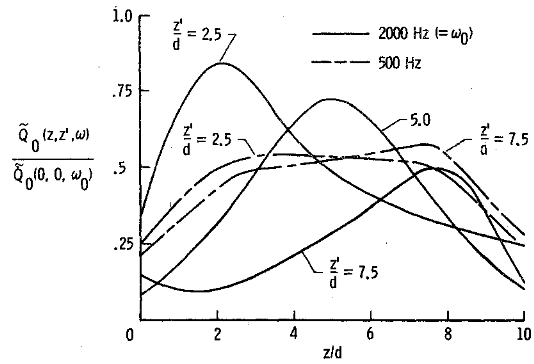

Fig. 5 Apparent source cross correlation at f = 500, 2000 Hz.

The results from the inversion of the source cross correlation $\tilde{Q}_0(z,z',\omega)$ for frequencies 500 and 2000 Hz are shown in Fig. 5 as functions of the jet downstream distance z/a, for a given fixed reference position (z'). The results obtained are consistent with more general results published in the literature. The lower frequencies are more evenly correlated over several jet diameters than the higher frequencies. In fact, the highest frequencies peak sharply at the origin with fastest decay occurring with the increases in spatial separation.

References

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